<u>AP Calculus – Summer Packet</u>

Welcome to Calculus!

Going into AP Calculus, there are certain skills that have been taught to you over the previous years that we assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

We assume that you have basic skills in Algebra. Being able to solve equations, work with algebraic expressions, factoring, for example should now be a part of you. If not, you would not be going onto AP Calculus. So topics that we think you absolutely need to know are included here. These are skills that are used continually in AP Calculus.

On the following pages are questions listed by topic. **Please do each problem <u>only</u> in the space provided. Additional pages will not be graded**. Work in pencil so you can make corrections. If you are unsure of how to attempt these problems, look at your notes, websites or talk to friends. Building a small study group from now that can be your support group next year as well is a good idea. Do not fake your way through these problems. Make sure you understand how to do all these by the end. As stated, students are notoriously weak in them, even students who have achieved well prior to AP Calculus.

This packet is due the first day back in school in the fall. It will be graded for completion not accuracy. Accuracy, and therefore learning, will be evaluated with an in-class test with select questions from the packet.

When handing in the packet, be sure your name appears on the first sheet and all sheets are stapled together. All work needs to be shown. Also, do not rely on your calculator. Remember, more than half of your AP exam next year is taken without a calculator. So try and do as much as you can without a calculator – even the graphs. Remember the graphs are just sketches with intercepts, asymptotes and end behavior being the most important.

It is a mistake to decide to do this now. Let it go until mid-summer. We want these techniques to be relatively fresh in your mind in the fall. Also, do not wait to do them at the very last minute. Give yourselves the last two weeks of vacation to do these.

AB student – please do not do the pages that say BC only.

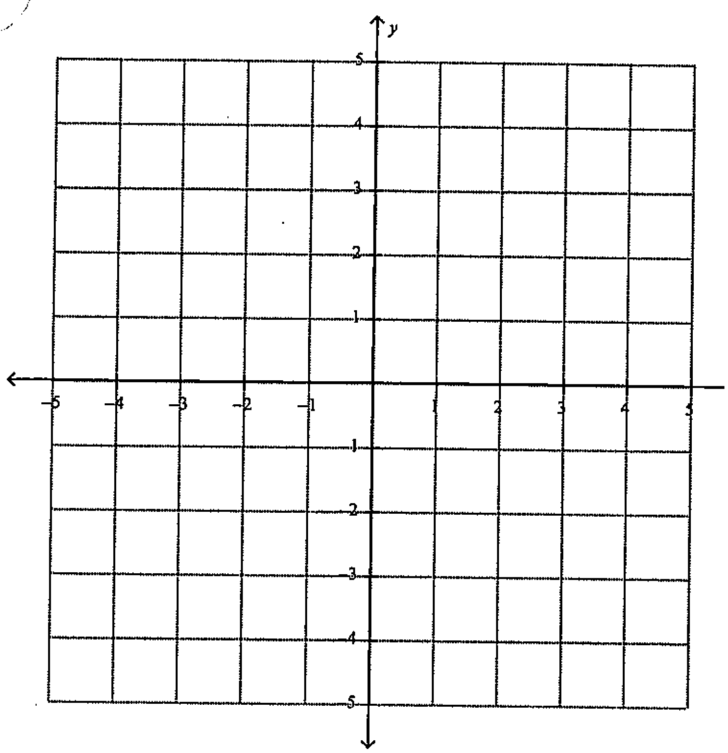
We look forward to seeing you in the next school year.

AP Calculus Teachers
Westhill High School

1. Graphs of the following functions will be used routinely in Calculus. Practice these graphs until you can demonstrate them from memory. You need to make a general sketch of each of these graphs and must know their domain, range and end-behavior.

All graphs w/o calc.

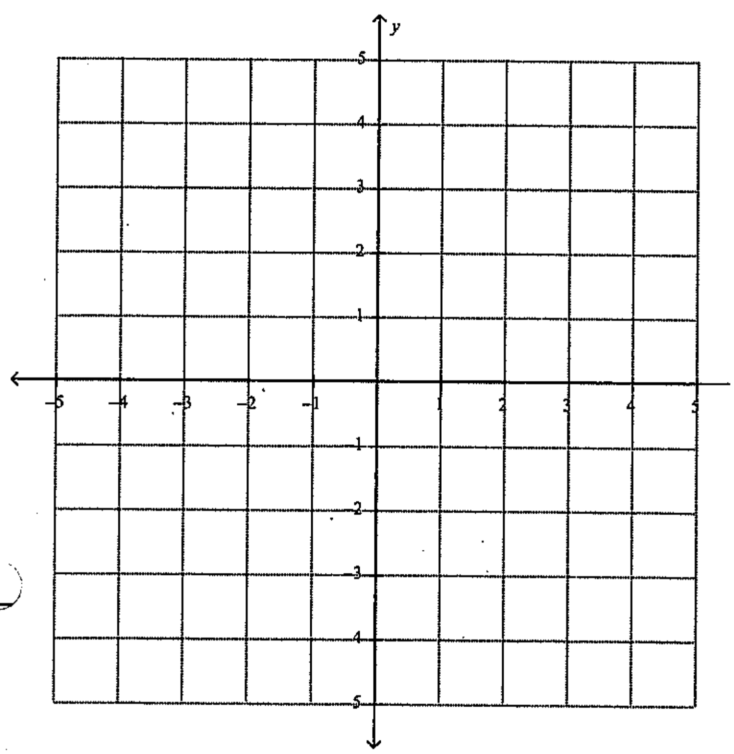
a) $y = -3x + 2$



Domain ______________________

End Behavior ______________________

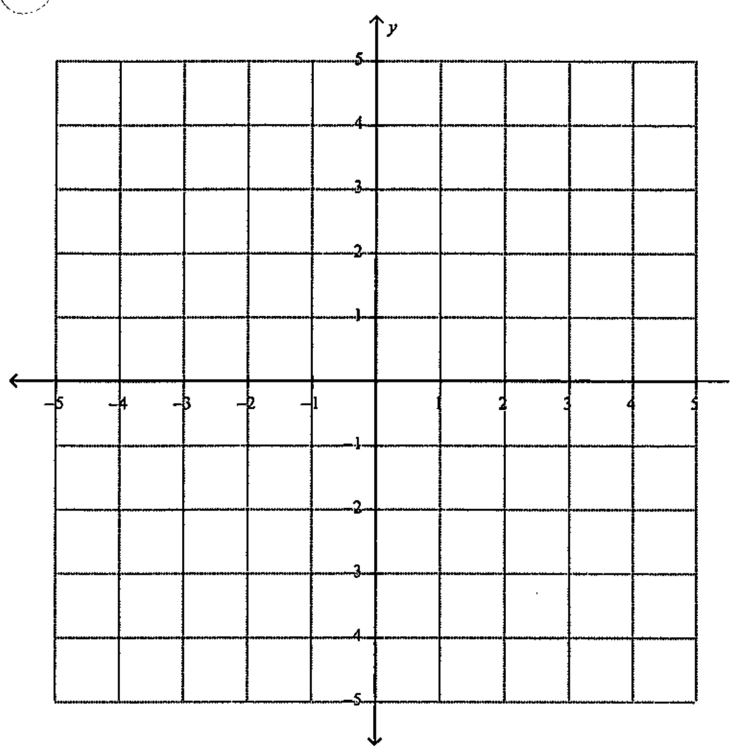
b) $y = \sqrt[3]{x-2} + 3$



Domain ______________________

End Behavior ______________________

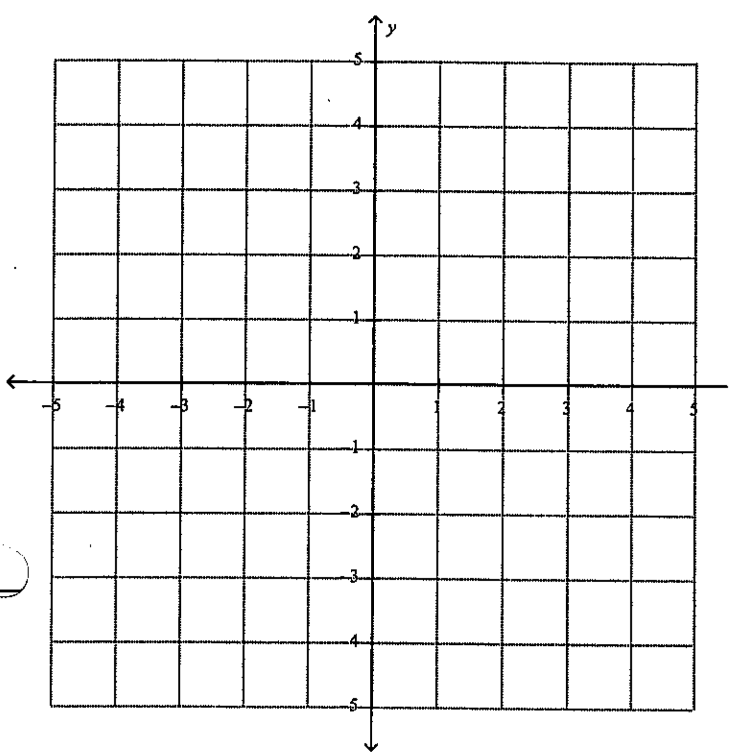
c) $y = e^{x} - 2$



Domain ______________________________

End Behavior __________________________

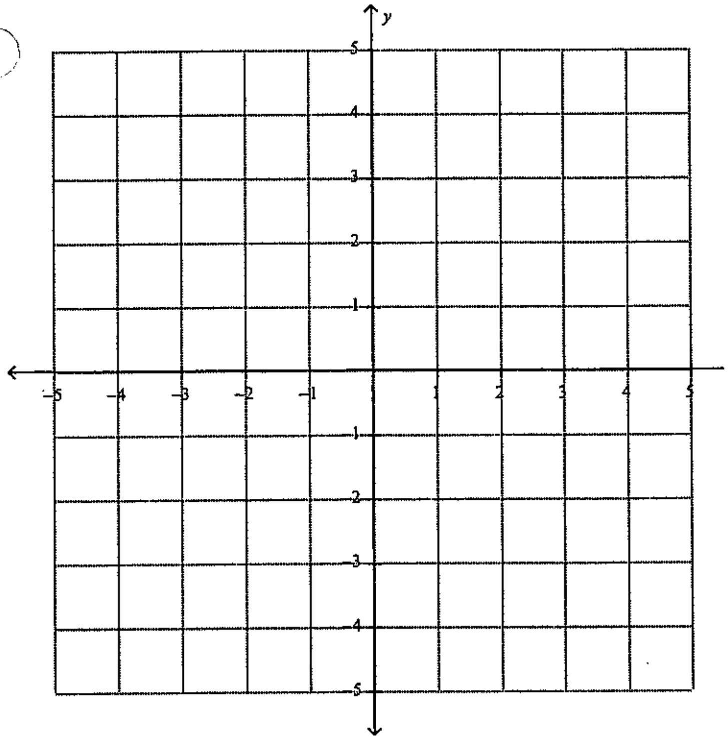
d) $y = (x-2)^{2} + 1$



Domain ______________________________

End Behavior __________________________

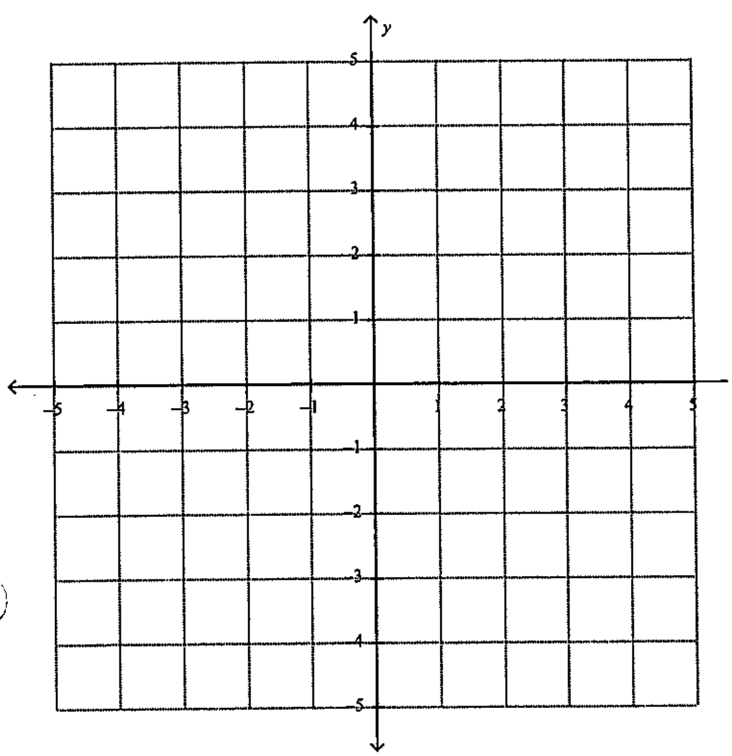
e) $y = \frac{1}{x}$



Domain ______________________________

End Behavior __________________________

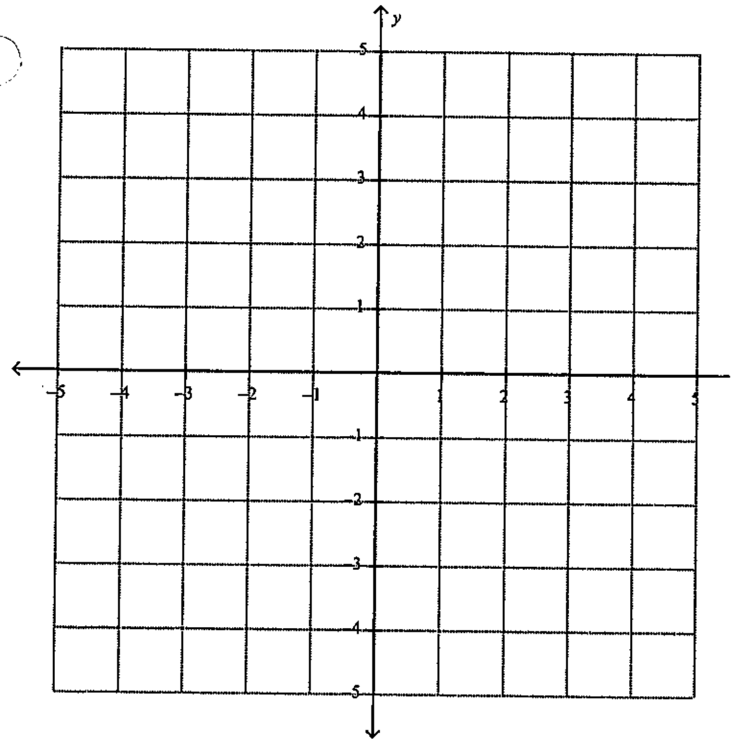
f) $y = \frac{1}{x^2}$



Domain ______________________________

End Behavior __________________________

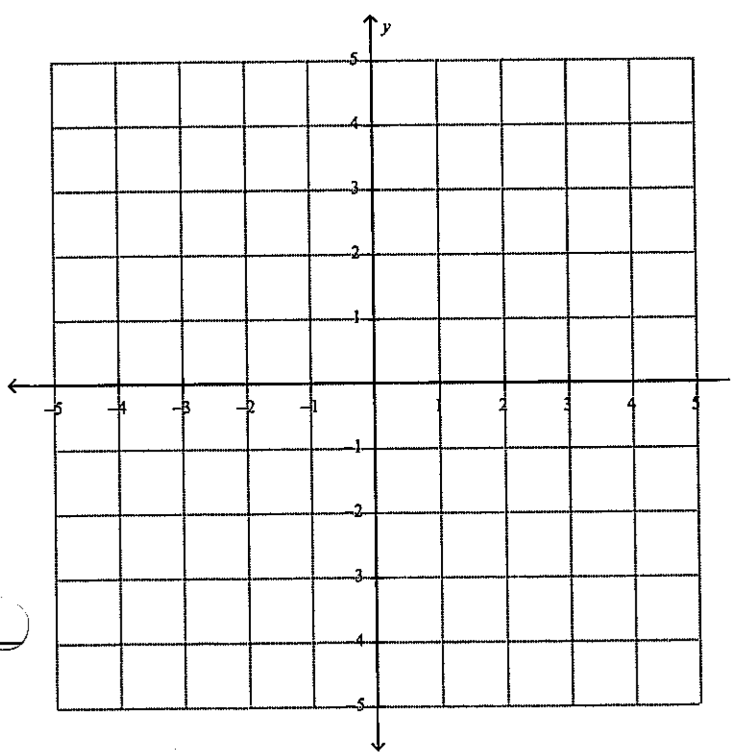
g) $y = (x-1)^3 + 1$



Domain ______________________

End Behavior ______________________

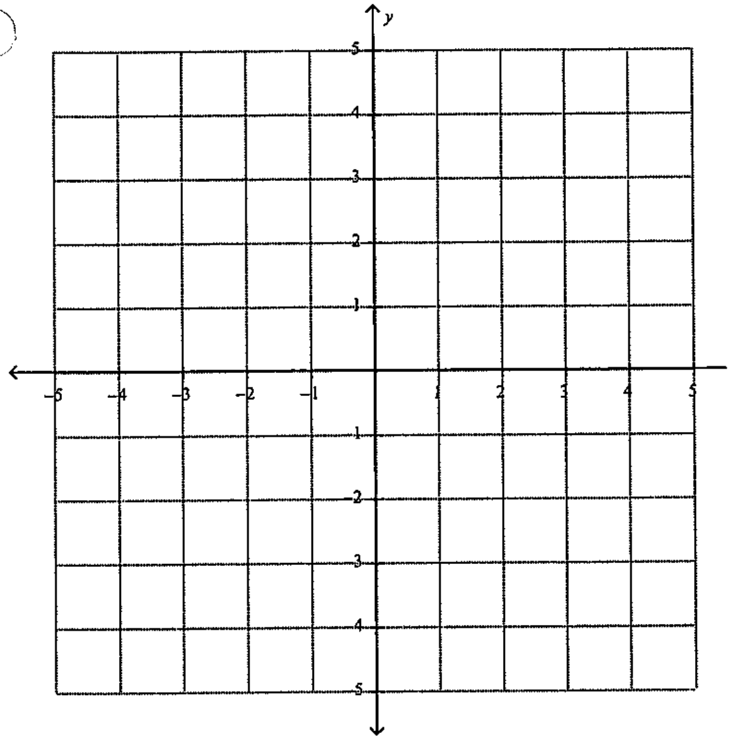
h) $y = |x+2| - 1$



Domain ______________________

End Behavior ______________________

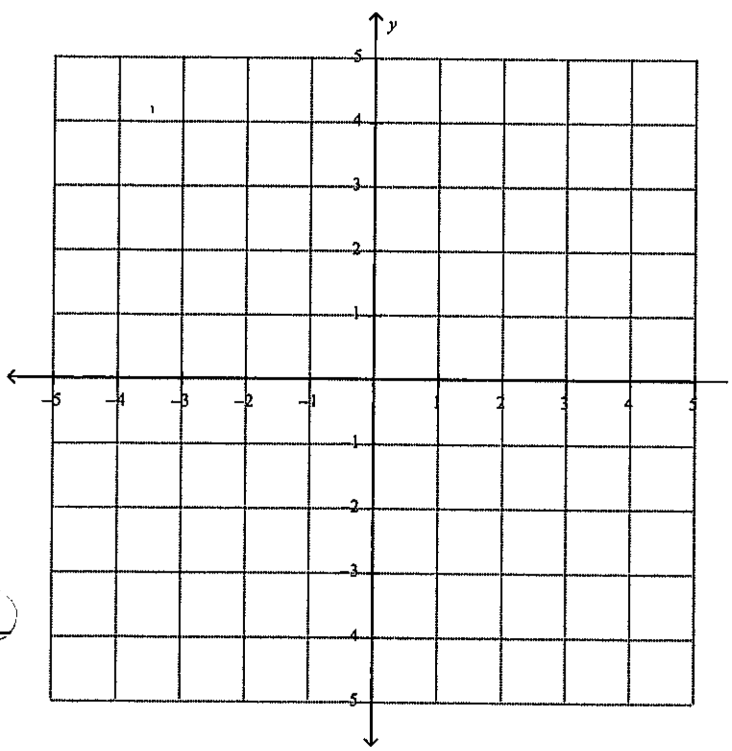
i) $y = 2\sqrt{x-2} + 1$



Domain ______________________

End Behavior ______________________

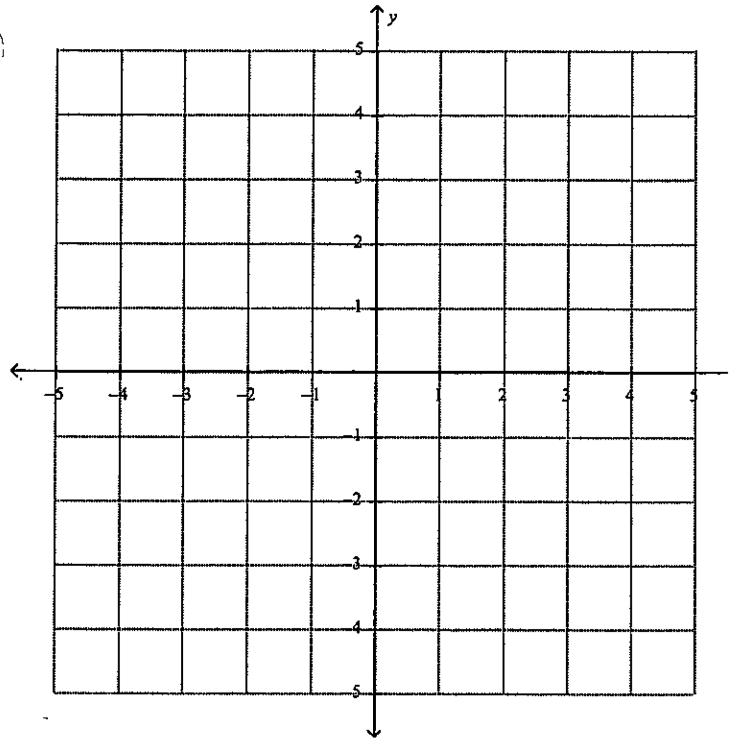
j) $y = \ln x$



Domain ______________________

End Behavior ______________________

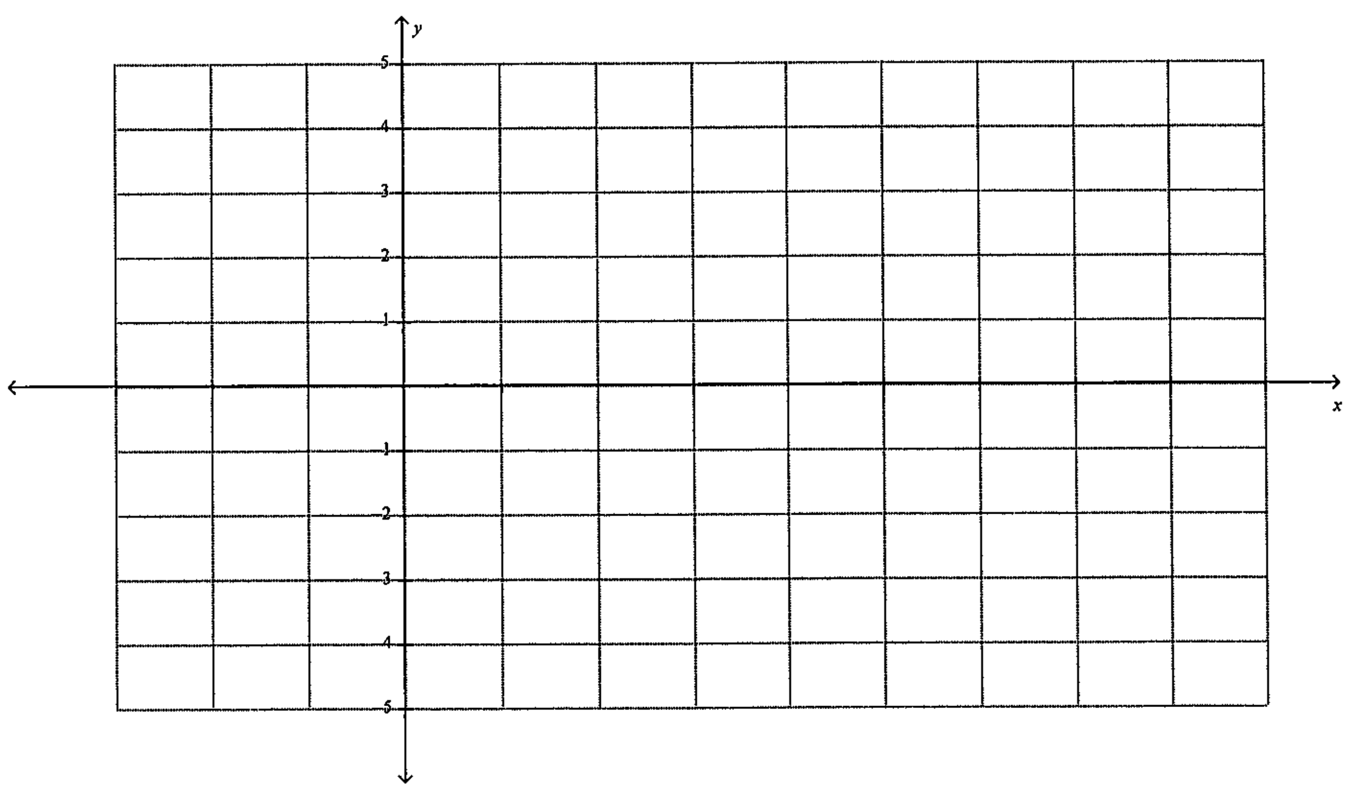
k) $y = \sqrt{9 - x^2}$



Domain ______________________

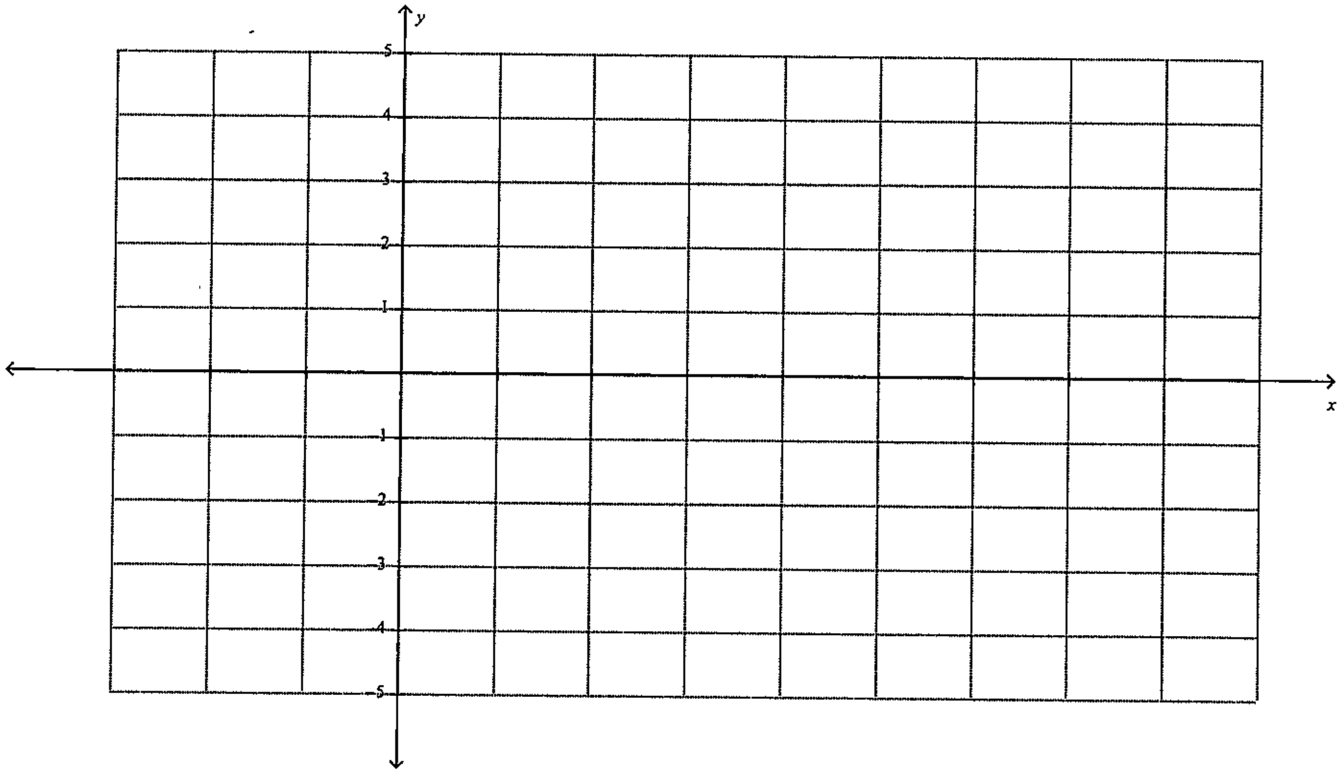
End Behavior ______________________

l) $y = \sin x$



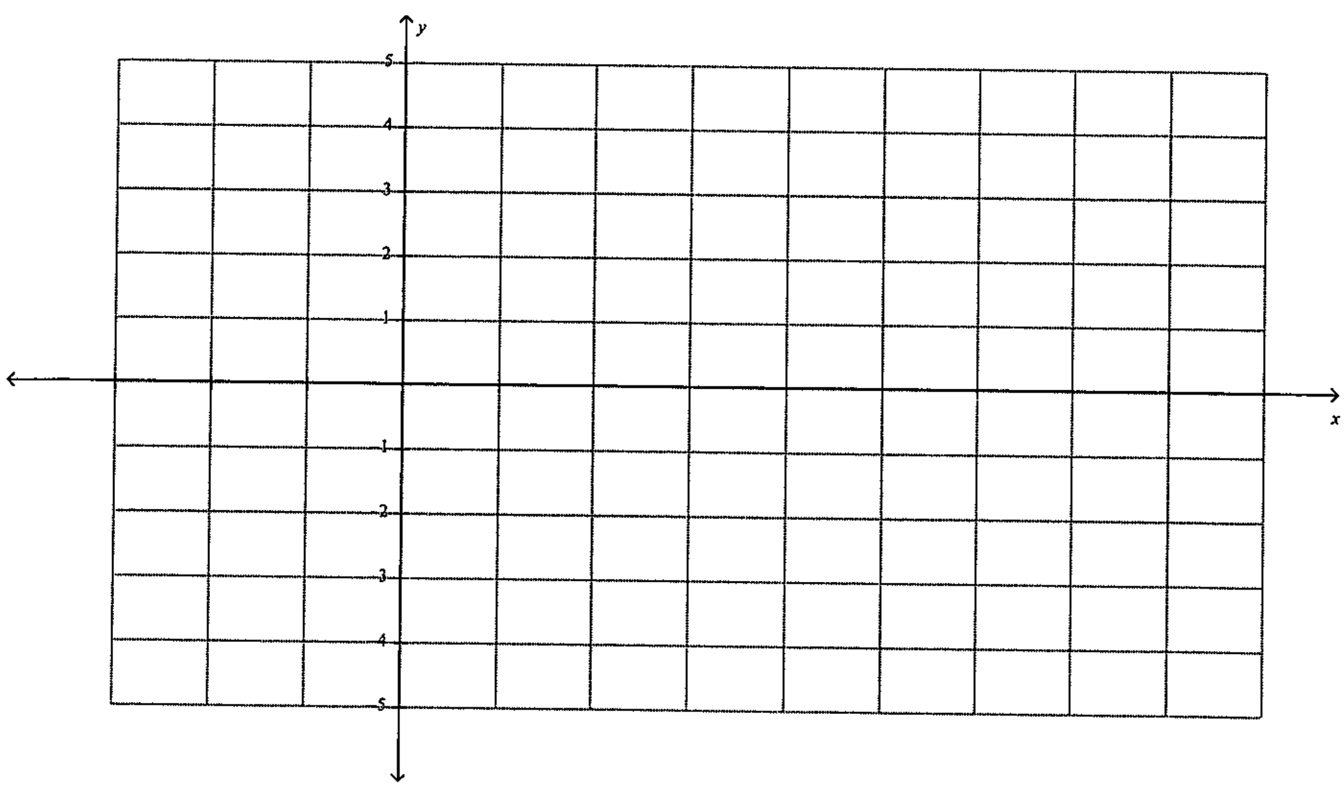
Domain ______________________

m) $y = 2\cos 3x$



Domain ______________________

n) $y = \tan x$

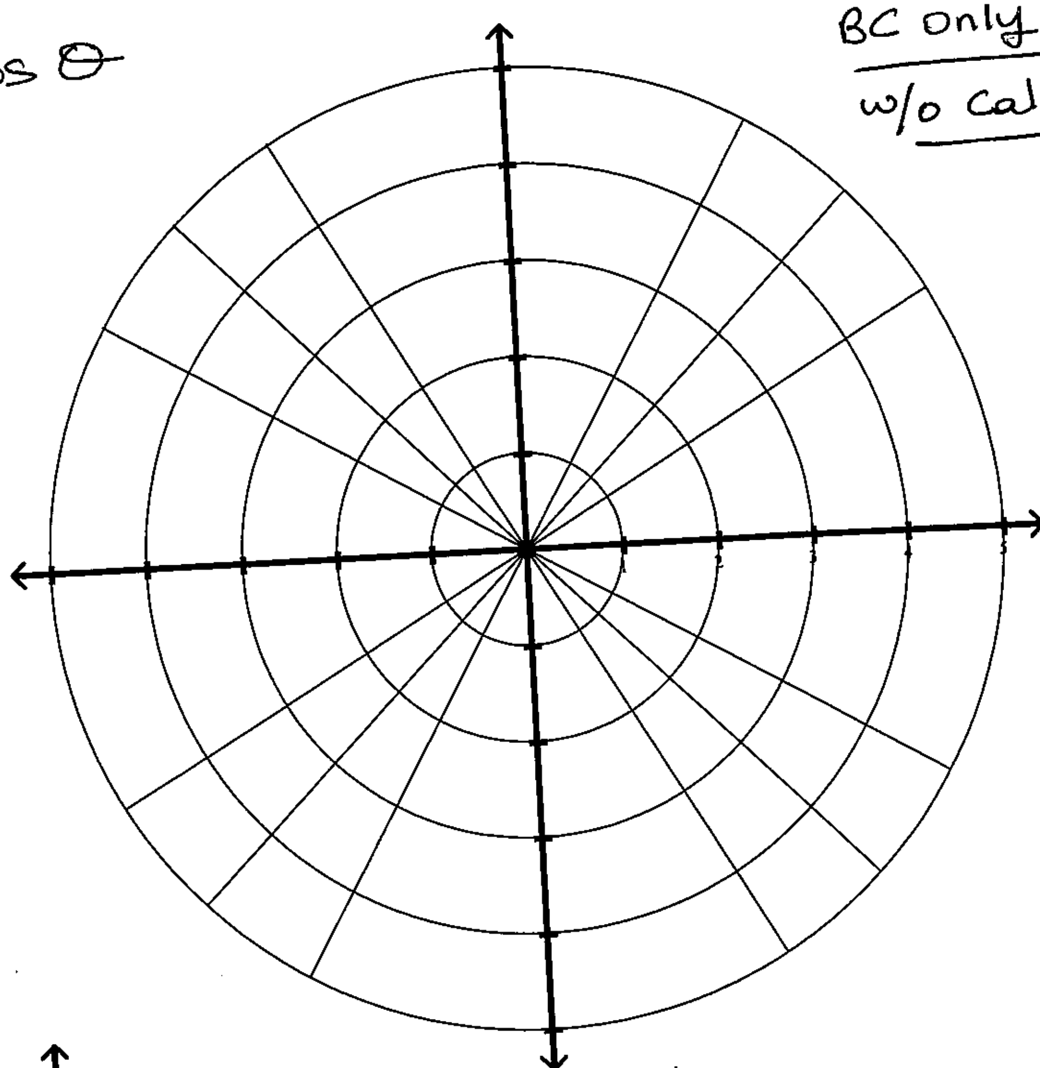


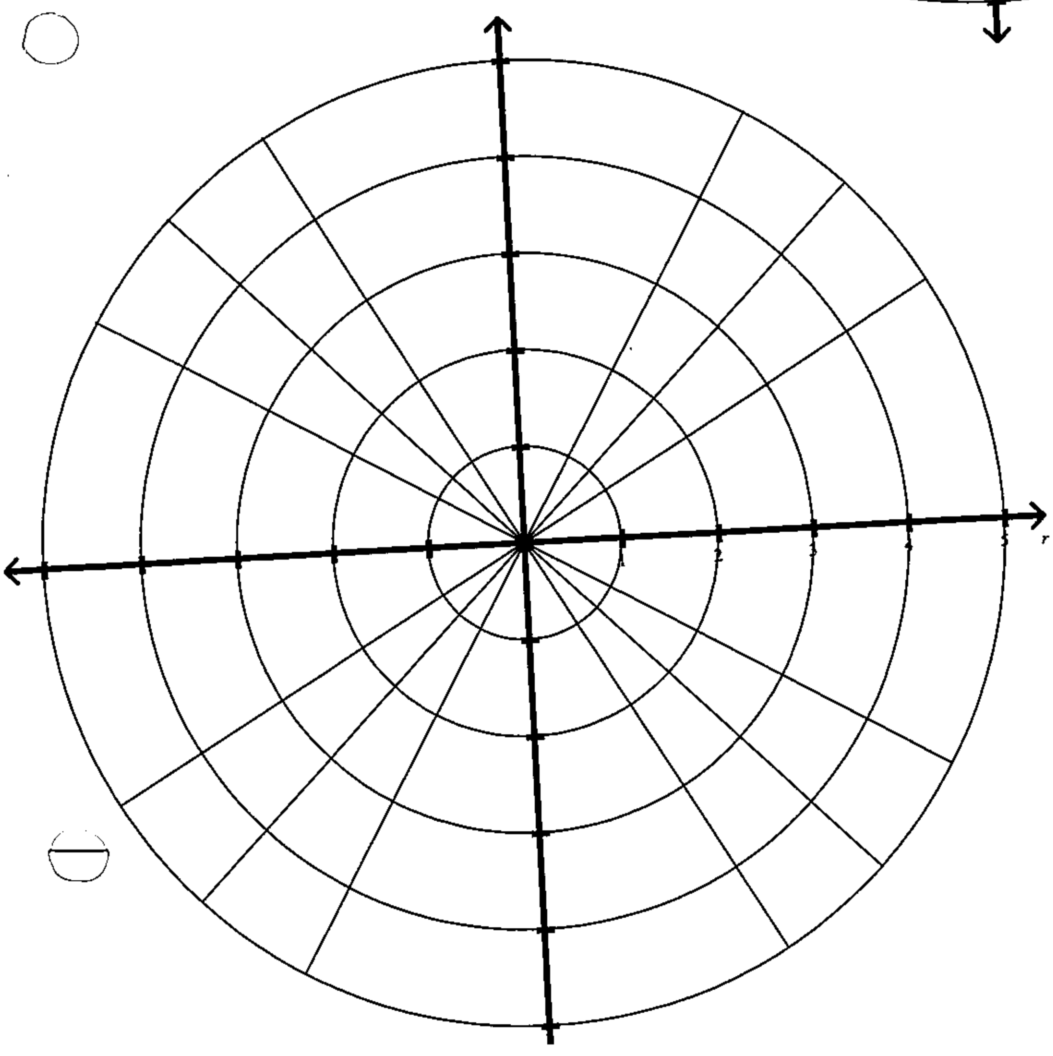
Domain ______________________

o) $r = 1 + 2\cos\theta$

BC only

w/o calc





p) $r = 2\sin 2\theta$

2. Solve the following equations and inequalities for x without using a calculator. Answers must be exact. No decimals.

a) $e^{2x} = 5$

b) $\ln(x+3) = 4$

c) $2\sin^2 x \cos x = \cos$ $\quad [0, 2\pi]$

d) $2\sin^2 x + 3\sin x - 2 = 0$ $\quad [0, 2\pi]$

e) $8^{3x+1} = 128^{x-2}$

f) $\log_2(x-2) + \log_2 x = 3$

g) $x + \frac{6}{x} = 5$

h) $4x(x-2) - 5x(x-1) = 2$

i) $2x^2 - (x+2)(x-3) = 12$

j) $x + \frac{1}{x} = \frac{13}{6}$

k) $x^4 - 9x^2 + 8 = 0$

l) $-12 \le \frac{5x-6}{2} \le -7$

3. Expand the following expressions using properties of logarithms.

(a) $\ln(x^2\sqrt{y})$

(b) $\log_3\left(\frac{x+3}{x^2}\right)$

4. Rewrite the expression $\log_5(x+3)$ into an equivalent expression using only natural logarithms.

5. Condense the following logarithmic expressions.

(a) $\log_7\left(\frac{1}{5}\right)+\log_7\left(\frac{5}{49}\right)$

(b) $\log_3 48 - 2\log_3 4$

(c) $e^{3\ln x}$

6. Solve for x.

a) $\ln e^3 = x$

b) $\ln e^x = 4$

c) $\ln x + \ln x = 0$

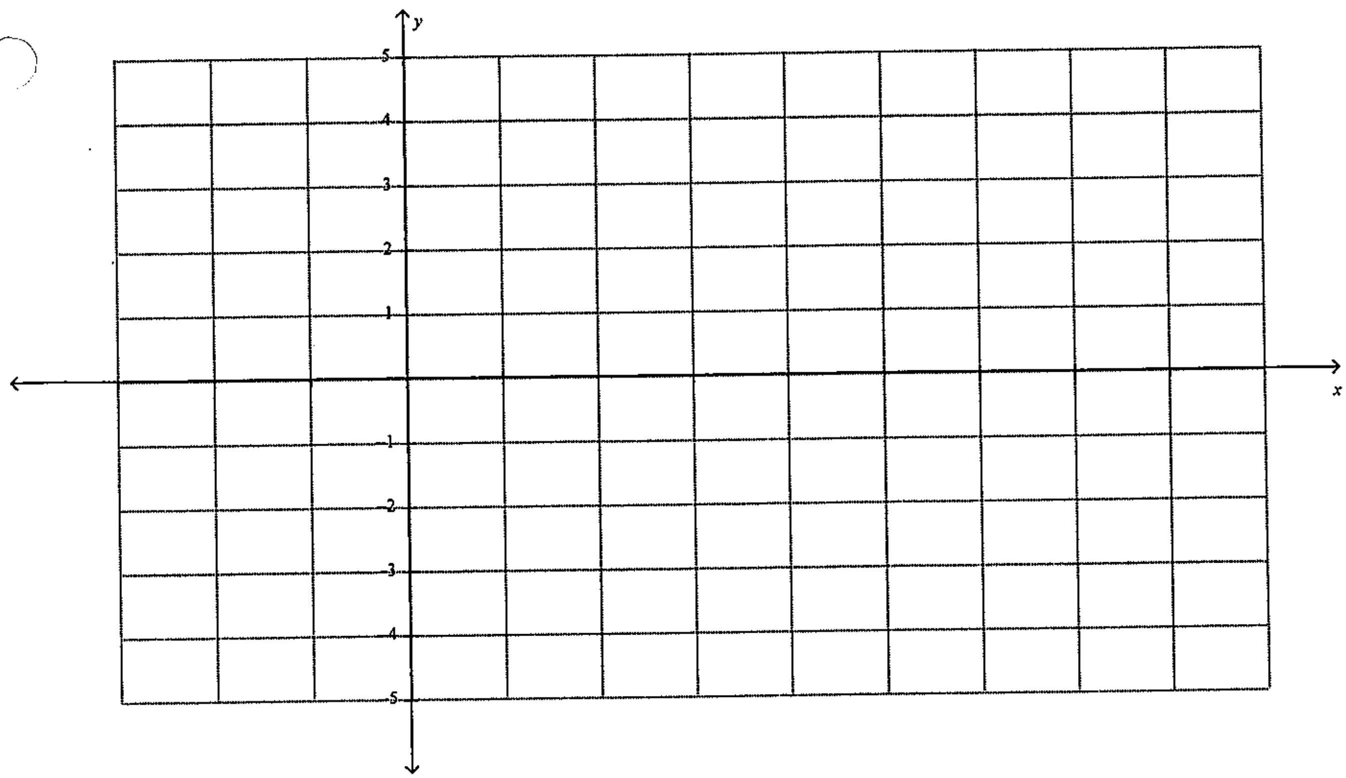
d) $e^{\ln 5} = x$

e) $\ln 1 - \ln e = x$

f) $\ln 6 + \ln x - \ln 2 = 3$

g) $\ln(x+5) = \ln(x-1) - \ln(x+1)$

7. Given $f(x)=4\sin\left(\frac{\pi}{2}x\right)-2$



(a) state the amplitude of the function:_______

(b) state the period of the function: _______

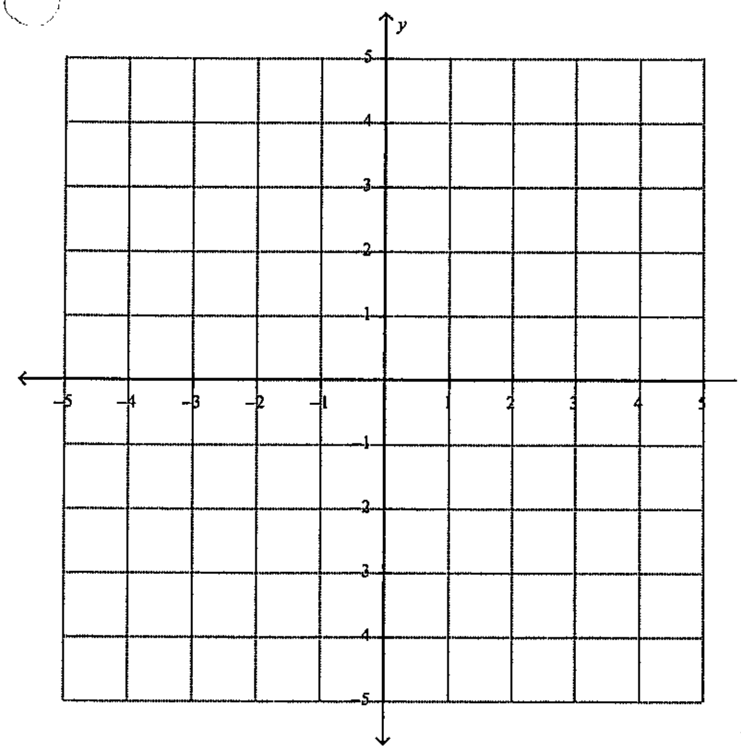
(c) state any vertical or horizontal shifts if there are any:

Vertical: ________ Horizontal: ________

(d) graph one period of this function on the axes provided. Clearly indicate your axes' scales.

(e) Write an equation of the line that contains the maximum point and minimum point of the period graphed.

8. Given $y = \dfrac{2(x+3)(x-2)}{(x-3)(x+3)}$



Identify the following:

(a) The coordinates of any holes/gaps: ____________________

(b) The equation(s) of any vertical asymptotes: ____________________

(c) The equation of the horizontal asymptote (if any): ____________________

(d) x-intercept(s) ____________________

(e) y-intercept(s) ____________________

(f) On the axes provided, graph the function to include all of the above.

9. Without using a calculator, evaluate the following giving the exact answer (no decimals).

(a) $sin\left(\frac{2\pi}{3}\right)$ (b) $cos\left(-\frac{5\pi}{4}\right)$ (c) $tan\left(\frac{11\pi}{6}\right)$ (d) $csc\left(\frac{7\pi}{6}\right)$

10. Simplify the following trigonometric expressions.

(a) $\frac{1-\cos^2 x}{\sec^2 x-1}$ (b) $\sec x \cdot \sin 2x$

11. Evaluate without using a calculator. Answers for g & h must be in radians.

a) $\cos 0$ b) $\sin 0$ c) $\tan\frac{\pi}{2}$ d) $\cos\frac{\pi}{4}$

e) $\sin\frac{\pi}{2}$ f) $\sin\pi$ g) $\arccos\frac{\sqrt{3}}{2}$ h) $\arctan 1$

i) $\tan(\arccos\frac{2}{3})$ j) $\cos\left[\sin^{-1}\left(-\frac{3}{5}\right)\right]$ k) $\cot\left[\arctan\left(\frac{5}{8}\right)\right]$

12. Find the solution of the equations for $0 \leq x < 2\pi$.

$2\sin^2\theta = 1 - \sin\theta$ $\quad$ $2\tan\theta - \sec^2\theta = 0$ $\quad$ $\sin 2\theta + \sin\theta = 0$

13. Circle the two expressions that are equivalent.

a) $\cos^2 x$ $\quad$ b) $(\cos x)^2$ $\quad$ c) $\cos x^2$

14. Circle the two expressions that are equivalent.

a) $(\sin x)^{-1}$ $\quad$ b) $\arcsin x$ $\quad$ c) $\sin x^{-1}$ $\quad$ d) $\frac{1}{\sin x}$

16. Are the following statements true? If not, explain why.

a) $\frac{2k}{2x+h} = \frac{k}{x+h}$ $\quad$ b) $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$ $\quad$ c) $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$

d) $3\frac{a}{b} = \frac{3a}{3b}$ $\quad$ e) $3\frac{a}{b} = \frac{3a}{b}$ $\quad$ f) $3\frac{a+b}{c} = \frac{3a+b}{c}$

17. Simplify.

a) $\dfrac{x/2}{x/4}$

b) $h \div \dfrac{(x+h)}{h}$

c) $\dfrac{\sqrt{x-2}+\dfrac{5}{\sqrt{x-2}}}{x-2}$

19. Solve the equation.

a) $4x^2-21x-18=0$

b) $x^3+3x^2-5x-15=0$

c) $x^4-9x^2+8=0$

21. Using your graphing calculator, graph the equation $y=x^3-x$ and answer the following questions.

a) Is the point (3, 2) on the graph?

b) Is the point (2, 6) on the graph

c) Is the function even, odd, or neither?

d) What is the y intercept?

e) Find the x intercepts.

f) State the open intervals on which the graph of y is increasing or decreasing. Use interval notation.

g) Find all relative extrema (max and min). State the x-coordinate and value of the function for each extremum.

22. Determine if the relation is even, odd, or neither. You must use $f(x)=f(-x)$ and $-f(x)=f(-x)$.

a) $f(x)=2x^2-7$

b) $f(x)=-4x^3-2x$

c) $f(x)=4x^2-4x+4$

23. Write the equation in point-slope form of the line that passes through the point $(2,4)$ and is parallel to the line $2x+3y-8=0$.

24. Find the equation of the line that is perpendicular to the line $2x + 3y - 8 = 0$ at the point (1,2)

25. The line with a slope of 5 that passes through the point $(-1,3)$ intersects the x axis at a point. What are the coordinates of this point?

26. What are the coordinates of the point at which the line passing through the points $(1,-3)$ and $(-2,4)$ intersects the y axis?

27. Given $f(x)=|x-3|-5$ find $f(1)-f(5)$.

28. Given $f(x)=x^2-3x+4$ find $f(x+2)-f(2)$.

29. Find the domain for each of the following functions algebraically. Write it using interval notation.

a) $h(x)=\dfrac{1}{4x^2-21x-18}$

b) $k(x)=\sqrt{x^2-5x-14}$

c) $p(x)=\dfrac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$

d) $y=\ln(2x-12)$

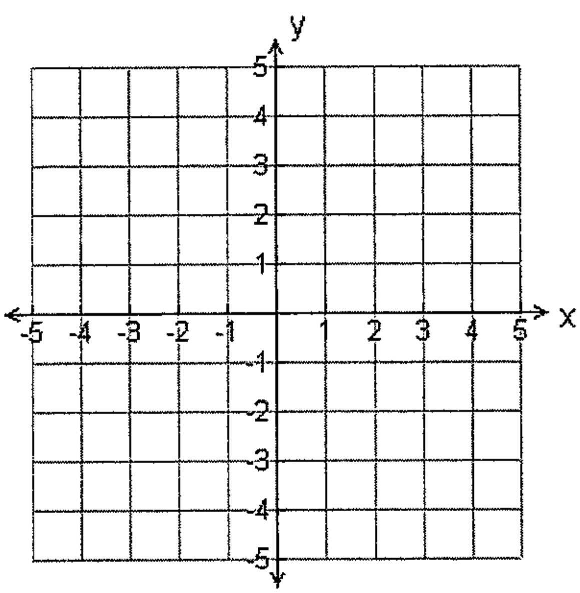
30. Find $f(x+h)$ for $f(x)=x^2-2x-3$.

31. Find $\dfrac{f(x+h)-f(x)}{h}$ if $f(x)=8x^2+1$

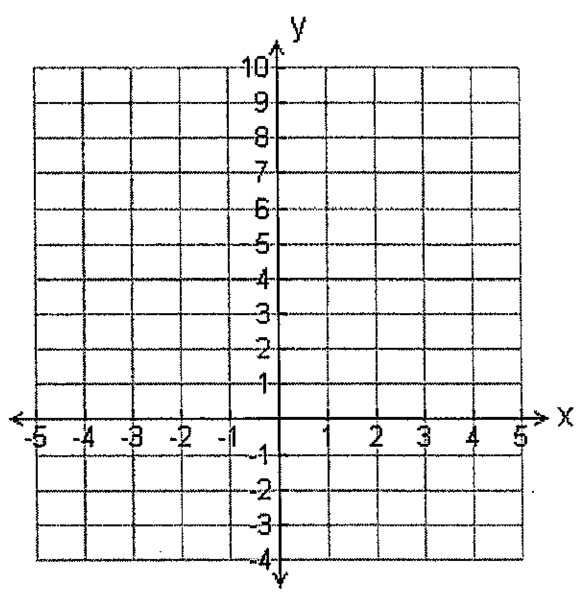
32. *Given* $f(x)=\dfrac{1}{x}$ *Find:* $\dfrac{f(x+h)-f(x)}{h}$

33. Sketch the graph of each function.

a) $f(x)=\begin{cases} 1 & x\le 0 \\ -1 & x>0 \end{cases}$



b) $f(x)=\begin{cases} 2x & (-\infty,-1) \\ 2x^2 & [-1,2) \\ -x+3 & (2,\infty) \end{cases}$



34. Given $f(x)=x-3$ and $g(x)=\sqrt{x}$ complete the following

a) $f(g(x))=$

b) $g(f(x))=$

c) $f(f(x))=$

35. Given $f(x)=\dfrac{1}{x-5}$ and $g(x)=x^2-5$ complete the following

a) $f(g(7))=$

b) $g(f(v))=$

c) $g(g(x))=$

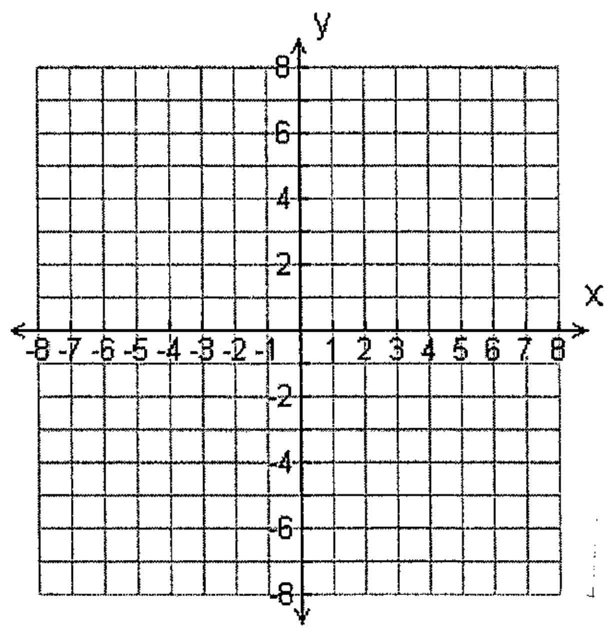
36. Let $f(x)=2x-2$. Complete the following:

a) Sketch the graph of $f(x)$.

b) Determine whether f has an inverse function.

c) Sketch the graph of $f^{-1}(x)$.

d) Give the equation for $f^{-1}(x)$



37. Simplify using only positive exponents. Do not rationalize the denominator.

a) $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

b) $\left(\dfrac{1}{x^{-2}}+\dfrac{4}{x^{-1}y^{-1}}+\dfrac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

38. If $f(x)=x^2-1$, describe in words what the following would do to the graph of *f(x)*.

a) $f(x)-4$

b) $f(x-4)$

c) $-f(x+2)$

d) $5f(x)+3$

e) $f(2x)$

f) $|f(x)|$

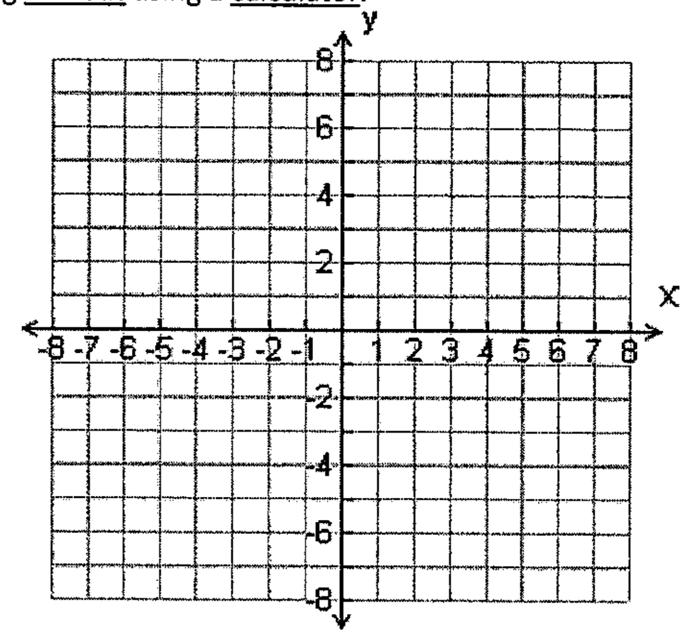
11. Let $f(x) = -1 + \ln(x - 2)$. Complete the following <u>without</u> using a <u>calculator</u>.

(a) Sketch the graph of f. State the equation of asymptotes and exact value of intercepts.

(b) Determine whether f has an inverse function. Justify your answer.

(c) Sketch the graph of $f^{-1}(x)$. State the equation of asymptotes and exact value of intercepts.

(d) Give the equation for $f^{-1}(x)$.



BC only

Polar Equations and Parametric Equations

1. Change from polar to rectangular form.

$(-2,\pi)$

2. Change from rectangular to polar form.

a. $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

b. $(8,-15)$

3. Change the rectangular equation to polar form.

a. $x^2+y^2+8x=0$

b. $x^2=5y$

4. Convert from polar form to an equation in rectangular form.

$r=4\sin\theta$

5. Obtain an equation in terms of x and y by eliminating the parameter. Then identify the curve.

a. $x=2t$, $y=t^2$

b. $x=2+2\sin\theta$, $y=3+2\cos\theta$

Vectors and Dot Products

1. Given $a = \langle 5, -12 \rangle$ **and** $b = \langle -3, -6 \rangle$. Find the following:

   a. **a + b**

   b. **2a + 2b**

   c. $|a - b|$

2. Find a unit vector that has the same direction as $-3i + 7j$.

3. What is the angle between the given vector and the positive direction of the x-axis?

   $i + \sqrt{3}j$

4. Determine whether the given vectors are orthogonal, parallel or neither.

   $a = \langle 4, 6 \rangle \qquad b = \langle -3, 2 \rangle$

5. Find the angle between the vectors. Give your answer using an exact expression (no decimals).

   $a = \langle -2, 5 \rangle \qquad b = \langle 5, 12 \rangle$

BC only

Sequences and Series

1. Write the first five terms of the sequence.

   a. $a_n = \sin\left(\frac{\pi n}{2}\right)$

   b. b. $a_n = \left(-\frac{1}{4}\right)^n$

2. Name the next two apparent terms of the sequence. Describe the pattern you used to find these terms.

   a. $5, 10, 20, 40, \ldots$

   b. b. $3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \ldots$

3. Simplify the ratio of factorials.

$$\frac{(2n-1)!}{(2n+1)!}$$

4. Find the sum, if possible.

   a. $\sum_{n=1}^{\infty} 2(0.9)^{n-1}$

   b. $\sum_{n=1}^{50} 5n+3$

   c. $\sum_{n=1}^{12} (n^3 - n + 2)$

BC only

1. Given the graph of f(x), answer the following:

(a) $\lim_{x\to -5^-} f(x) =$ (b) $\lim_{x\to -5^+} f(x) =$

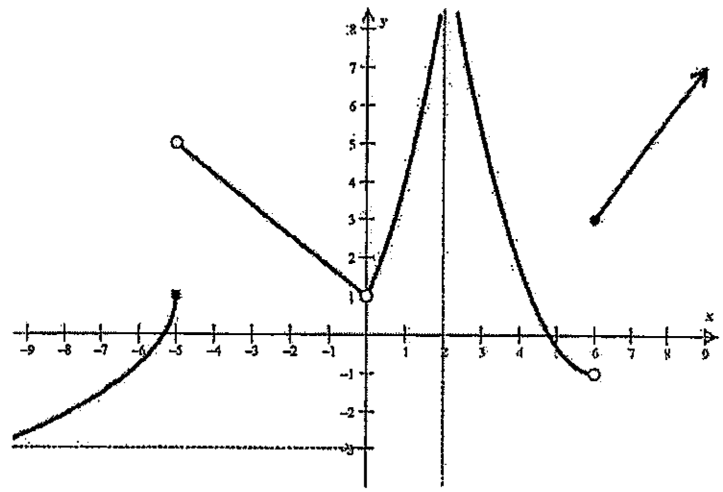
(c) $\lim_{x\to -5} f(x) =$ (d) $\lim_{x\to 0^-} f(x) =$

(e) $\lim_{x\to 0^+} f(x) =$ (f) $\lim_{x\to 0} f(x) =$

(g) $f(0) =$ (h) $\lim_{x\to 6^-} f(x) =$

(i) $\lim_{x\to 6^+} f(x) =$ (j) $\lim_{x\to 6} f(x) =$

(k) $\lim_{x\to -\infty} f(x) =$ (l) $\lim_{x\to \infty} f(x) =$



2. Evaluate the following limits analytically:

(a) $\lim_{x\to 0} \dfrac{x^4 - 7x^2}{x^3 + 3x^2} =$

(b) $\lim_{x\to 9} \dfrac{x-9}{3-\sqrt{x}} =$

(c) $\lim_{x\to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x-2} =$

d) $\lim_{x\to 4^-} \dfrac{x-1}{x-4}$

e) $\lim_{x\to 4^+} \dfrac{x-1}{x-4}$

f) $\lim_{x\to 4} \dfrac{x-1}{x-4}$

(g) $\lim_{x\to \infty} \left(\dfrac{5+2x}{3-x}\right) =$

(h) $\lim_{x\to -\infty} \left(\dfrac{x+4}{\sqrt{9x^2-2x+5}}\right) =$

(i) $\lim_{x\to \infty} \left(\dfrac{4x+7x^3}{2x^3-x^2+3}\right) =$

BC only

Equation of Tangent Line

Find the equation of the tangent line for the following two functions (using the limit definition of the derivative) at the given point.

a) $f(x)=2x^2+3x$ at $(1,5)$

b) $f(x)=\sqrt{x+1}$ at $x=3$